PHYS 320 ANALYTICAL MECHANICS

Dr. Gregory W. Clark

Fall 2108

TODAY

Newton's Laws

Review of statics

Statics: next steps … trusses

NEWTON'S LAWS

Valid only in INERTIAL reference frames!

I Every body continues in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by forces impresses upon it.

II The change of motion is proportional to the motive force impresses; and is made in the direction of the line in which that force is impressed.

Note: these two refer to a specific body (mass)

Newton's Second Law

- Suppose $F_{net} = ma \neq constant.$
- Suppose $F_{net} = F(x)$:

$$F(x) = m\,a = m\,\dot{x} = m\frac{dv}{dt} = m\frac{dv}{dx}\frac{dx}{dt} = mv\frac{dv}{dx}$$

then $F(x)\,dx = m\,v\,dv$ ← Separation of variables!

$$\int_{x_i}^{x_f} F(x)\,dx = \int_{v_i}^{v_f} m\,v\,dv = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \Delta K$$

WORK-ENERGY THEOREM!

Newton's Second Law

- Suppose $F_{net} = ma \neq constant.$
- Suppose $F_{net} = F(v)$:

$$F(v) = m\,a = m\,\dot{x} = m\frac{dv}{dt}$$

then $$dt = m\,\frac{dv}{F(v)}$$ ← Separation of variables!

$$\Delta t = \int_{v_i}^{v_f} \frac{dv}{F(v)}$$

Static Equilibrium

- Conditions of equilibrium:

$$\vec{F}_{net} = \sum_i \vec{F}_i = 0$$

2nd Law

$$\vec{\Gamma}_{net} = \sum_i \vec{\Gamma}_i = 0$$

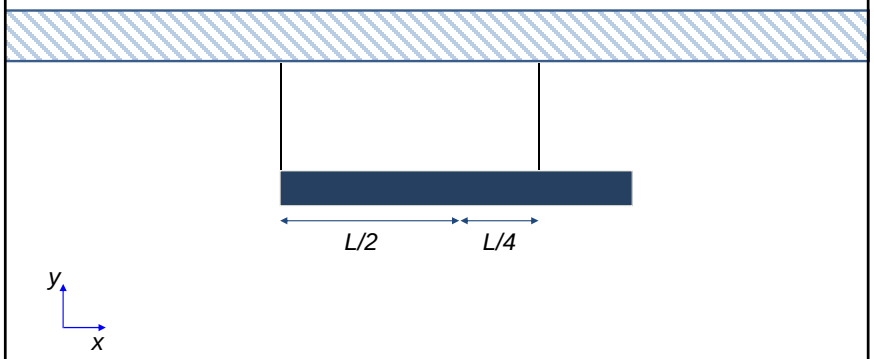
$$\vec{\tau}_{net} = \sum_i \vec{\tau}_i = 0$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

3rd Law

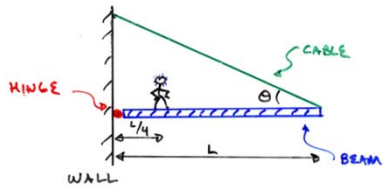
Static Equilibrium

Consider a plank of mass *M* and length *L* suspended by two strings as shown. Find the tension in each string:



Static Equilibrium

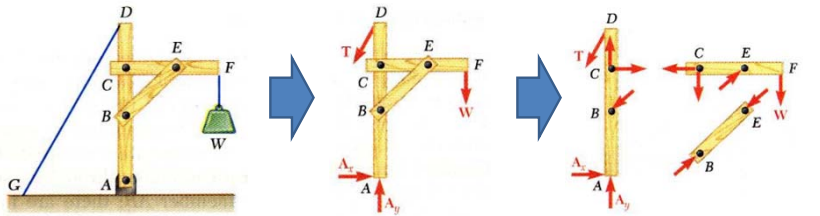
EX: A UNIFORM HORIZONTAL BEAM (W = 200N, L = 8m) IS ATTACHED TO A WALL VIA A HINGE. THE FAR END IS SUPPORTED BY A CABLE THAT MAKES AN ∠ OF 53° W.R.T. THE HORIZONTAL. IF A 600N PERSON STANDS 2 m FROM THE WALL, FIND THE TENSION IN THE CABLE AND THE FORCE EXERTED ON THE BEAM BY THE WALL ("REACTION FORCE").



Static Equilibrium: Structures

- Simple systems of multiple components:
  - **Trusses**: formed from *two-force* **members**
  - **Frames**: contain at least one **member** with > 2 forces
  - **Machines**: structures containing moving parts designed to transmit and modify forces.

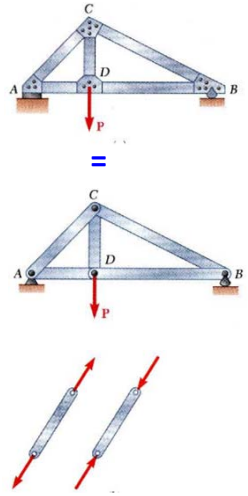
Example of a **frame**



Ref: Beer & Johnston, Mechanics for Engineers: Statics

Static Equilibrium: Structures

- **Trusses**:
  - A truss consists of straight, two-force **members** connected at joints; no member is continuous through a joint.
  - Most structures are made of several trusses joined together to form a 3D framework. Each truss carries those loads which act in its plane and may be treated as a 2D structure.
  - Bolted or welded connections are modeled as pinned together; forces acting at member ends reduce to a single force.
  - If forces tend to pull the member apart, it is in **tension**. If the forces tend to compress the member, it is in **compression**.
  - Truss members assumed to have negligible mass (compared to the loads they carry).



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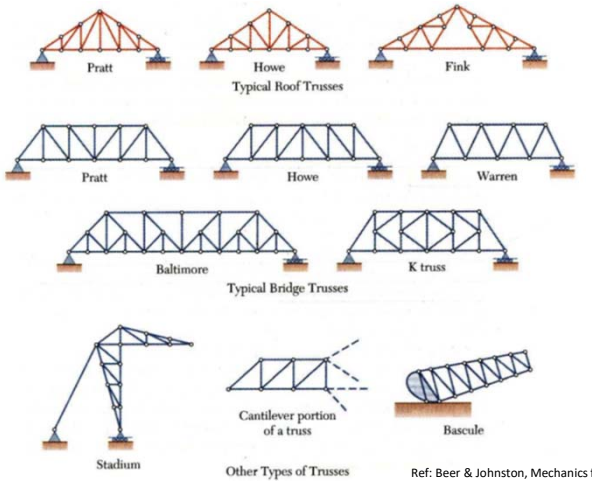
Static Equilibrium: Structures

- Some sample **trusses**:



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